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METHODS AND PROSPECTS OF THE DIRECT EXPERIMENTAL VERIFICATION AND REFINEMENT OF THE "PACKET" MODEL OF EXTERNAL HEAT TRANSFER IN A FLUIDIZED BED

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Analysis of the Basis of the Model

The packet model proposed by Mickley [1] has served as a basis for explaining a whole series of special characteristics of external heat transfer in fluidized beds and for the construction of engineering formulas facilitating the practical estimation and calculation of heat-transfer coefficients. After considering new experimental data as to the structure of the boundary zone [2, 3] and the sharp criticism of the packet model made by Syromyatnikov [4], a more detailed analysis of the fundamental principles of the model has now become a matter of great importance.

A fluidized bed of solid particles agitated by a rising gas flow is usually very inhomogeneous, not only in the boundary zone, but also over the whole volume of the apparatus. The local porosity ε fluctuates constantly from $\varepsilon = 1$ to $\varepsilon = \varepsilon_{min} \approx 0.4$. In order to describe a number of phenomena associated with heat and mass transfer and catalytic reactions, many research workers [5] prefer to consider these fluctuations schematically and (by way of simplification) to assume that at any specific instant the fluidized bed consists of regions existing in one of two limiting states: $\varepsilon = 1$, i.e., gas bubbles free from particles, and $\varepsilon = \varepsilon_{min}$, constituting the so-called dense or compact phase (packets). The basis for making such a far-reaching schematization when analyzing catalytic reactions in a fluidized bed is the vast quantitative (practically qualitative) difference in the properties of these limiting states. Inside the bubble the gas never encounters catalyst grains, and no reaction occurs. In the dense phase, however, the reaction rate reaches a maximum. To a first approximation this so-called two-phase model of the fluidized bed gives a satisfactory explanation for the reduction in the yield of the reaction in a fluidized bed by comparison with a stationary catalyst and also reveals the main factors capable of influencing the degree of yield. Later on, however, when attempting to refine the quantitative laws of the process [6], it was found necessary (to a certain extent) to allow for intermediate states as well, namely, particles spilling down into the interior of the bubble, the "tail" of the bubble, and the "cloud" of adjacent particles undergoing vigorous gas-exchange with the bubble itself. It may well be that a description of the processes based on a fuller account of all the continuously varying states existing between the bubble and the packet will lead to major advances [7].

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An analogous rough schematization of the structure of the boundary layer was proposed by Mickley [1] in analyzing external heat transfer: first a packet and then a bubble alternately approaches the wall (or a heat-transfer surface immersed in the fluidized bed), and the heat-transfer intensity fluctuates over a wide range. A similar concept was developed by Zabrodskii [8] in considering the coming and going of the neighboring layer of particles relative to the wall.

The existence of such a very transient mechanism of external heat transfer was qualitatively confirmed by Mickley [9] and others [10, 11], who immersed a low-inertia heater with a low heat capacity (a foil) into the bed and noted the sharp temperature fluctuations of the latter. It was considered that after the arrival of a packet the foil rapidly lost temperature, and on replacing the packet by a gas bubble (owing to the sudden reduction in heat outflow) the temperature of the foil increased.

The well-known approximate Mickley formula derived from these simplified considerations:

$$\bar{\alpha} = \sqrt{(1-f_0)\frac{4}{\pi}\lambda_e c_T \rho_T (1-\varepsilon_0)\varphi} \tag{1}$$

was later supplemented by Baskakov [12] who introduced the so-called "contact" resistance of the gas interlayer between the wall and the packet.*

An analysis of this approximate relationship provided a good explanation for a number of qualitative characteristics of external heat transfer in fluidized beds [13]. Thus, in view of the fact that the effective thermal conductivity of the dense layer $\lambda_e = \lambda_g f(\varepsilon_0)$ was directly proportional to the thermal conductivity of the gas λ_g, it was easy to understand the approximate dependence of the heat-transfer coefficient on the nature of the gas $\alpha \sim \sqrt{\lambda_g}$. The fact that the volumetric heat capacity of solids $c_T\rho_T$ depends only very slightly upon their chemical nature (since approximately the same number of atoms always occurs in unit volume) enables us to explain why the heat-transfer coefficient α is almost independent of the grain material. Since the frequency of the pulsations and the interchange of bubbles and packets in the fluidized bed is in order of magnitude equal to $\varphi \approx 1/2\pi\sqrt{q/H}$, while the height of the bed in both laboratory columns and industrial systems usually only varies over the range $H \approx 0.1$–1 m, it is easy to understand why $\bar{\alpha}_{max}$ is almost identical, despite the great variety of conditions. The dependence of $\bar{\alpha}$ on the diameter d of the particles in the fluidized bed [which does not enter directly into Eq. (1)] is, in fact, very weak. Finally, quantitative estimates [13] of $\bar{\alpha}_{max}$ based on Eq. (1) yield excellent agreement with experiment. On the basis of Eq. (1) we deduced a relationship between the maximum external heat-transfer coefficient and the physical characteristics of the gas and grains:

$$\bar{\alpha}_{max} = 3.27\frac{\lambda_g^{0.5}\nu_g^{0.154}(c_T\rho_T)^{0.5}}{d^{0.231}}\left(\frac{\rho_g}{\rho_T}\right)^{0.077} \text{W/(m}^2\cdot{}^\circ\text{K)}, \tag{2}$$

which agreed satisfactorily with experiment and was similar to Zabrodskii's empirical formula [8].

Naturally, under practical conditions the limiting states ($\varepsilon = 1$ and $\varepsilon = \varepsilon_0$) are accompanied by all the intermediate states $\varepsilon_0 < \varepsilon < 1$. Let us consider how the thermal characteristics of the boundary layer (thermal conductivity λ, volumetric heat capacity $c\rho$, thermal diffusivity $a = \lambda/c\rho$, and thermal assimilation $s = \sqrt{\lambda c\rho}$) are likely to vary in these intermediate states. The latter two parameters characterize processes of transient heat propagation in the medium under consideration, while the quantity $s = \sqrt{\lambda c\rho}$ enters directly into Eq. (1), which gives the average heat-transfer coefficient in the presence of transient heating of the medium close to the wall, and $\bar{\alpha} \sim s$.

For $\varepsilon = 1$ we have a pure gas with its own values of λ_g $(c\rho)$ g, a_g, and s_g. In the second limiting state $\varepsilon = \varepsilon_0$ the thermal conductivity of the loose granular bed $\lambda_e = \lambda_g f(\varepsilon_0) \approx 10\lambda_g$ and is practically independent of the particle material [14]. The volumetric heat capacity of the dense granular bed $(c\rho)_e = c_T\rho_T(1-\varepsilon_0) \approx 10^3 (c\rho)g$. Hence $a_e \approx 10^{-2} a g$, and $\alpha_e \approx 100\alpha g$. An approximate curve indicating the way in which these parameters vary with increasing volumetric concentration of the solid phase $(1-\varepsilon_0)$ is presented in Fig. 1. We see from the fig-

*Antonishin [17] also proposed allowing for transience in the initial stage of heat transfer between the gas and the particle.

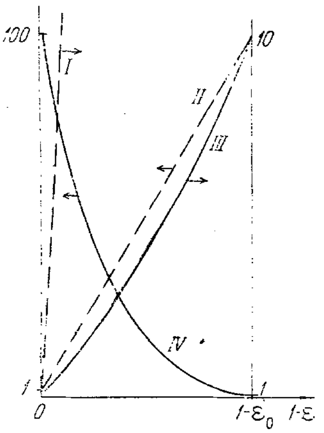

Fig. 1. Change in the thermal characteristics of a dispersed medium with increasing proportion of the solid phase. I) $c\rho/(c\rho)_g$; II) α/α_g; III) λ/λ_g; IV) a/a_g.

ure that the thermal characteristics of the limiting states (bubbles and packets) differ by 1-2 orders of magnitude; this is the *main justification* for considering the process of heat transfer to the continuously varying intermediate states of the boundary zone (at any rate, to a first approximation) schematically, taking account of simply the extreme states (as in the packet model).

This schematic packet model should certainly be improved and refined at a later date by allowing for all the intermediate states also. However, in criticizing the model for imprecision [4], we must not forget the great service which it has performed in practical engineering [13] and treat it as simply untrue.

The foregoing estimates provide a simple basis for introducing the concept of contact resistance [12]. The time τ of the transient heating of a layer of thickness b is given by the approximate relationship

$$\tau \approx 0.1\frac{b^2}{a}. \tag{3}$$

Thus, the heating time of a compact granular bed $\tau_e = 0.1(b^2/a_e)$ is 100 times longer than that of a gas layer of the same thickness $\tau_g = 0.1(b^2/a_g) = 0.1(b^2/100a_e) = \tau_e/100$. In other words, whereas the heat penetrates into the packet fairly slowly and in a transient manner, the gas interlayer between the packet and the wall is heated almost instantaneously, and heat transfer through the layer proceeds on an almost steady-state basis, with a linear temperature drop and a constant thermal resistance $R_k = b/\lambda_g$.

Inverting Eq. (3),

$$b = \sqrt{10a\tau}, \tag{4}$$

we obtain yet another interesting estimate. For a thermal diffusivity of the compact granular bed $a_e \approx 2\cdot10^{-7}$ m²/sec and a mean pulsation frequency of 5 Hz, i.e., $\tau \approx 0.2$ sec, it follows from Eq. (4) that the depth of heating of the packet during this time will be approximately $6\cdot10^{-4}$ m = 0.6 mm. This estimate shows that for fine grains with diameter $d \approx 0.1$-0.3 mm the Mickley packet model describes the process more closely [1], while for coarser particles with $d \approx 0.5$-1.0 mm the Zabrodskii model is preferable [8].

Heat Transfer from a Low-Inertia Heater to a Fluidized Bed

Equation (1) was obtained for an average heat-transfer coefficient $\bar{\alpha}$ defined as the ratio of the average thermal flux q_0 taken over many fluctuations to the *constant* temperature drop θ_0 between the wall and the core of the fluidized bed. Such constancy of the heater surface temperature θ_0 = const, however, is only possible for an infinitely great thermal inertia, such as is practically achieved under real industrial conditions.

In order to detect the fluctuations in the heat flow perfectly clearly during the mutual exchange of bubbles and packets at the surface, Mickley [9] proposed using a low-inertia heater (a platinum foil), on which fluctuations in the outgoing thermal flux should appear as recordable fluctuations in the temperature of the actual heater $\theta(\tau)$. Mickley gave no detailed analysis of the way in which these fluctuations might be affected by the thermal

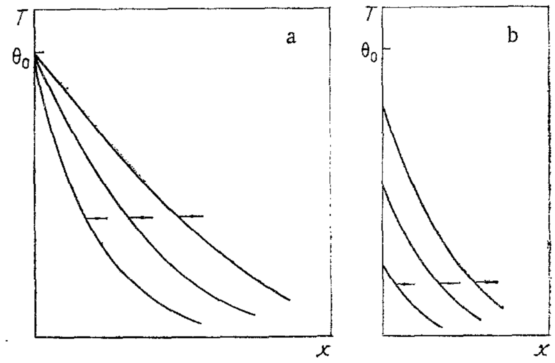

Fig. 2. Time dependence of the temperature distribution inside a packet: a) heater with a great inertia; b) heater with a very low inertia.

characteristics of the heater itself and its thermal history prior to the arrival of the cold packet. It was tacitly assumed that the low-inertia heater was simply a convenient indicator, while the character of the changes taking place in the thermal flux which it conveyed to the packet $q(\tau)$ should remain the same as in the case of a wall with a very great thermal capacity. This presumption led to a number of inaccuracies and paradoxes, especially when it was desired [2] to measure and describe the temperature fluctuations of the heater itself and the structure of the boundary zone at its surface at the same time.

In order to elucidate the influence of the thermal characteristics of the actual heater on heat exchange with the packet we shall now carry out a corresponding calculation. We shall regard the problem as one-dimensional. Close to the left-hand side of the plane $x = 0$ is a heater of thickness δ with a heat capacity per unit area $(c\rho\delta) = C_1$ and a high thermal conductivity. An external source (e.g., an electric current passing through the foil) evolves a certain quantity of heat q_0 = const per unit surface area of the heater in unit time. Owing to the great thermal conductivity, the temperature θ at all points inside the heater may be regarded as identical and depends solely on time $\theta = \theta(\tau)$. At the initial instant $\tau = 0$, when the heater is characterized by a certain temperature θ_0, a cold packet falls on it from the right, having a constant temperature $T(x, \tau)$ over the whole half-plane $0 < x < \infty$; we shall arbitrarily set $T(x, 0) = 0$. The heat from the heater is propagated by simple heat conduction in the packet, and the temperature of the latter at subsequent moments of time satisfies the equation

$$\frac{\partial T}{\partial \tau} = a_e \frac{\partial^2 T}{\partial x^2}. \tag{5}$$

At the boundary with the heater the medium assumes the heater's temperature,

$$T(0, \tau) = \theta(\tau). \tag{6}$$

The thermal flux entering into the packet $q(\tau) = -\lambda_e(\partial T/\partial x)_{x=0}$ is related to the thermal balance of the actual heater by the equation

$$C_1 \frac{d\theta}{d\tau} = q_0 - q(\tau) = q_0 + \lambda_e \frac{\partial T}{\partial x}\bigg|_{x=0}. \tag{7}$$

Before solving the complete system of equations (5)-(7) with specified initial conditions, let us consider the limiting cases of a heater with an infinitely great thermal inertia ($C_1 \to \infty$), corresponding to ordinary industrial conditions, and also a completely inertialess heater ($C_1 \to 0$).

In the first case $C_1 = \infty$, it follows from (7) that $d\theta/d\tau = 0$ and $\theta = \text{const} = \theta_0$. Equation (5) with the boundary condition $T(0, \tau) = \theta_0$ has a well-known [15] solution for a half-space; using this solution we may calculate the thermal flux from the wall:

$$q'(\tau) = \theta_0 \sqrt{\frac{\lambda_e(c\rho)_e}{\pi\tau}} \tag{8}$$

at an arbitrary moment of time τ after the start of the process and the instantaneous heat-transfer coefficient:

$$\alpha'(\tau) = \frac{q'(\tau)}{\theta_0} = \sqrt{\frac{\lambda_e(c\rho)_e}{\pi\tau}}. \tag{8*}$$

In the second limiting case $C_1 = 0$, it follows from (7) that $q(\tau) = \text{const} = q_0$. The temperature of the inertia-free heater falls instantaneously from θ_0 to zero when the packet arrives and then starts rising together with the temperature of the adjacent zone of the packet, in accordance with the equation [15]

$$\theta'' = \frac{2q_0\sqrt{\tau}}{\sqrt{\pi\lambda_e(c\rho)_e}}. \tag{9}$$

Correspondingly, the formal instantaneous heat-transfer coefficient

$$\alpha''(\tau) = \frac{q_0}{\theta''(\tau)} = \sqrt{\frac{\pi}{4}\frac{\lambda_e(c\rho)_e}{\tau}} \tag{9*}$$

varies in accordance with the same law as the heater with infinite inertia, but exceeds it by a factor of $\sqrt{\pi^2/4} = 1.57$ times. Figure 2a and b shows that the temperature distribution and heating of the packet vary with time in completely different ways in these two limiting cases.

In the most general case, in which C_1 is neither equal to zero nor infinity, the solution of the system (5)-(7) may easily be obtained by an operational method for both $T(x, \tau)$ and $\theta(\tau)$. Transforming from the images to the original in the latter expression, we obtain

$$\theta(\tau) = \frac{q_0C_1}{\lambda_e(c\rho_e)}\left\{\sqrt{\frac{4}{\pi}\frac{\lambda_e(c\rho)_e\tau}{C_1^2}} + \exp\left[\frac{\lambda_e(c\rho)_e\tau}{C_1^2}\right]\times\right.$$
$$\left.\times\operatorname{erfc}\left[\sqrt{\frac{\lambda_e(c\rho)_e\tau}{C_1^2}}\right] - 1\right\} + \theta_0\exp\left[\frac{\lambda_e(c\rho)_e\tau}{C_1^2}\right]\operatorname{erfc}\left[\sqrt{\frac{\lambda_e(c\rho)_e\tau}{C_1^2}}\right]. \tag{10}$$

We may also correspondingly calculate the thermal flux,

$$q(\tau) = q_0 - C_1\frac{d\theta}{d\tau}.$$

In order to analyze the resultant solution (10), we should really take a more convenient time scale, not containing the parameter C_1 (variable in the foregoing analysis). A suitable scale of this kind is

$$\tau^* = \lambda_e(c\rho)_e\frac{\theta_0^2}{q_0^2} = \frac{\lambda_e(c\rho)_e}{\alpha_0^2}, \tag{11}$$

where $\alpha_0 = q_0/\theta_0$ coincides in order of magnitude with the average heat-transfer coefficient from the wall to the fluidized bed. Let us then introduce the dimensionless time

$$\eta = \frac{\tau}{\tau^*} = \frac{\alpha_0^2\tau}{\lambda_e(c\rho)_e} \tag{12}$$

and the dimensionless parameter

$$M = \frac{C_1\alpha_0}{\lambda_e(c\rho)_e}, \tag{13}$$

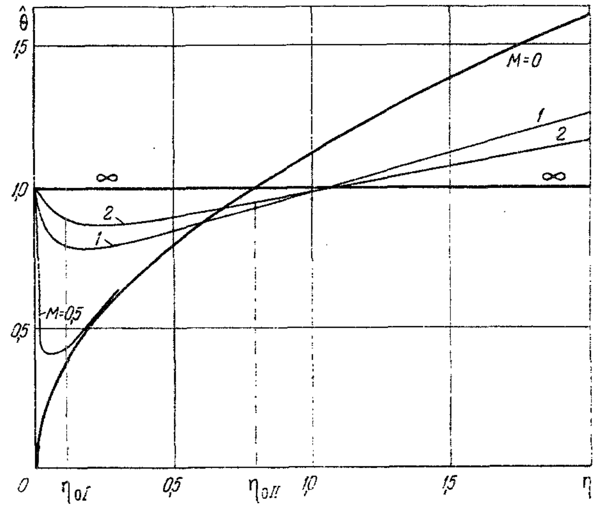

Fig. 3. Relative heater temperature as a function of its time of contact with the packet for various values of the thermal-inertia parameter M.

characterizing the relative inertia of the heater. Our solution (10) in the dimensionless variables η and $\hat{\theta}(\tau) = \theta(\tau)/\theta_0$ then takes the form

$$\hat{\theta}(\eta) = M\left\{\sqrt{\frac{4}{\pi}\,\frac{\eta}{M^2}} + e^{\eta/M^2}\operatorname{erfc}\sqrt{\frac{\eta}{M^2}} - 1\right\} + e^{\eta/M^2}\operatorname{erfc}\sqrt{\frac{\eta}{M^2}}. \quad (14)$$

The foregoing extreme cases may be derived from this general expression by putting $M \to 0$ and $M \to \infty$. Figure 3 presents a set of curves described by Eq. (14) both for the limiting values of the parameter M and for several intermediate M values of the order of unity. The latter are extremely typical: when a cold packet approaches, a real low-inertia sensor does in fact start cooling rapidly. However, as indicated by Baskakov [18], after a certain time $\tau_{min} = \eta_m \tau^*$ the closest layers of the packet are heated so much that the heater starts emitting less heat and its temperature begins rising together with the packet.

Thus, the character of the recorded heater-temperature/time curve $\theta(\tau)$ depends very considerably on the relationship between the calculated value of τ_{min} and the time τ_0 during which the packet remains close to the heater, after which it is replaced by the bubble. If $\tau_0 = \tau_{0I} < \tau_{min}$, then while the packet is in contact with the heater the temperature of the latter will in fact fall, as indicated by all research workers who have measured it. The situation will be quite different for very low-inertia heaters, for which τ_{min} is small and $\tau_0 = \tau_{0II} > \tau_{min}$. In this case the heater temperature will initially fall; however, in the interval $\tau_{min} < \tau < \tau_{0II}$ the packet will still be in contact with the heater, but the temperature of the latter will start rising, and this may be erroneously interpreted as meaning the departure of the packet and its replacement by the bubble. The possibility of such a relationship $\tau_0 > \tau_{min}$ has already been observed [16]. On replacing the packet by a cold bubble the characteristic time scale τ^* will, however, shorten by four orders of magnitude, and the heater temperature will start rising almost at once.

Since the foregoing analysis and the curves of Fig. 3 show that η_m depends on the parameter $M = [C_1/\lambda_e(c\rho)_e]\cdot(q_0/\theta_0)$ which contains not only the specific heat of the heater C_1, but also the initial temperature θ_0, the relationship between τ_0 and τ_{min} may still depend very considerably on the latter, i.e., on the preceding thermal history of the heater.

A more detailed analysis of Eq. (14) shows that for large values of the parameter $M > 10$, i.e., for a high thermal inertia of the heater C_1 and a low degree of heating of the latter θ_0 at the instant of arrival of the packet, the quantity η_m tends to a constant value $1/\pi = 0.318$ and $\tau_{min} = (1/\pi)\tau^* = 0.318[\lambda_e(c\rho)_e]/\alpha_0^2$. However, on reducing the thermal inertia C_1 or increasing θ_0, the quantity η_m and the time required to reach the minimum temperature τ_{min}, i.e., the transition from cooling to heating, is sharply reduced, and for $M < 0.1$ we shall have η_m M/2, i.e., $\tau_{min} \approx (M/2)\tau^* = (C_1/2)(\theta_0/q_0)$. Since the heater temperature θ_0 at the instant of arrival of the packet may vary considerably according to the period of contact between the heater and the bubble, the recorded temperature curves $\theta(\tau)$ corresponding to two successive packets may be completely different, even when their period of contact with the wall τ_0 is exactly the same.

Let us make some quantitative estimates.

For a granular bed of sand with a bulk density ρ_b = 1500 kg/m³ we have

$$\lambda_e(c\rho)_e \approx 36\cdot 10^4 \frac{J^2}{m^2\cdot sec\cdot {}^\circ K} \quad (15)$$

Putting

$$\alpha_0 \approx 400 \frac{J}{m^2\cdot sec\cdot {}^\circ K}, \quad (16)$$

we find that

$$\tau^* = \frac{\lambda_e(c\rho)_e}{\alpha_0^2} \approx 2.25 \text{ sec and } \tau_{min} = \eta_m \tau^* \approx 0.72\ . \quad (17)$$

Thus, for a very inertial heater under practical industrial conditions $\tau_0 \ll \tau_{min}$ the calculation of the average value of $\bar{\alpha}$ by Eq. (1) is quite permissible.

For Mickley's platinum foil of thickness δ = 25 μ = $25\cdot 10^{-6}$ m the thermal inertia

$$C_1 = 72 \frac{J}{m^2\cdot {}^\circ K} \quad (18)$$

and the parameter

$$M_1 = \frac{C_1\alpha_0}{\lambda_e(c\rho)_e} \approx 0.08 \ll 1. \quad (19)$$

In this case the time to reach minimum temperature is

$$\tau_{min} \approx \frac{C_1}{2\alpha_0} \approx 0.09\, c, \quad (20)$$

i.e., it would really be possible to find cases in which $\tau_0 > \tau_{min}$, and the foil starts heating before the packet has been replaced by the bubble.

It is considerably more difficult to estimate the parameters in the cases of Baskakov [16] and Syromyatnikov [2, 3]. Although the thickness of the foil in these cases was much smaller than $\delta \approx 5$ μ = $5\cdot 10^{-6}$ m, in contrast to the experiments of Mickley the foil was bonded to a continuous solid substrate, the thermal conductivity of the latter being an order of magnitude greater than the effective thermal conductivity of the packet. However, the thermal flux passing into this substrate was never measured, and an exact calibration of the consequently changing thermal inertia of the heater C_1 is therefore impossible, since the flow of heat into the substrate depends on the whole thermal prehistory of the latter.

Experimental Ways of Verifying and Refining the Packet Model

The foregoing calculations may be refined by allowing for the contact resistance R_k of the gas interlayer between the packet and the heat-transfer surface. However, a refinement of this kind will not make any qualitative difference to the foregoing conclusions. Since the mere measurement and recording of the temperature of a low-inertia heater will not enable us to assess the presence or absence of a packet in the immediate neighborhood of the heat-transfer surface uniquely and accurately, the method proposed by Mickley [9] is inconclusive and other experimental possibilities must accordingly be discussed and tested. One of these ways (the most promising in our own opinion) involves dispensing with the low-inertia heater and passing to a heater in which the surface temperature will remain practically constant: θ_0 = const. To this end the fluctuating thermal flux $q(\tau)$ must be measured with a low-inertia thermal sensor attached to the heater surface. In order to ensure that the temperature drop θ_0^0 in this type of thermal sensor, with thickness b and thermal conductivity λ, should not exceed 2% of the total drop θ_0 between the wall and the core of the fluidized bed, it is essential that the thermal resistance of the thermal sensor $R_T = b/\lambda$ should be $0.02/\alpha_0$, i.e.,

$$\frac{\lambda}{b} \approx 20000 \frac{J}{m^2 \cdot sec \cdot {}^\circ K} \tag{21}$$

If the thickness of the thermal sensor is $b = 0.1$ mm $= 10^{-4}$ m, this requires that its thermal conductivity should be $\lambda = 2$ J/m·sec·°K, which is characteristic of dense materials. For the heating time (3) to be no more than 0.01 sec, the thermal diffusivity of the material should be

$$a = 10\,b^2 = 10^{-7}\,m^2/sec, \tag{22}$$

whence the volumetric heat capacity of the material

$$c\rho = \frac{\lambda}{a} = 2 \cdot 10^7 \frac{J}{m^3 \cdot {}^\circ K}. \tag{23}$$

As the specific heat of solids $c \approx 10^3$ J/(kg·°K), a density of the material equal to $\rho \approx 20{,}000$ kg/m^3 is quite reasonable.

Since experimental prospects of recording rapid changes in the structure of the boundary layer have recently advanced very substantially, the development and construction of low-inertia thermal sensors would enable the instantaneous fluctuations in the thermal flux $q(\tau)$ to be recorded at the same time as these changes. The setting up of such experiments will certainly be very helpful in creating a more perfect model of the mechanism underlying the external heat transfer of a fluidized bed. This model should make a more accurate allowance for the stochastic character of the continuously varying circumstances in the boundary zone and the role of the intermediate states between those of the "packet" and "bubble" types.

NOTATION

ε, $\varepsilon_0 = \varepsilon_{min}$, local porosity and porosity in the packet; λ, λ_T, λ_g, λ_e, thermal conductivities of the medium, the grains, the gas, and the packet; c, c_T, c_g, C_1, specific heat (general), heat capacities of the grains and the gas, and unit area of the heater; ρ, ρ_T, ρ_g, ρ_e, densities of the medium, the grains, the gas, and the packet; a, a_T, a_g, a_e, thermal diffusivities of the medium, the grains, the gas, and the packet; α, $\bar{\alpha}$, α_{max}, α_0, α', α'', heat-transfer coefficients — general, average, maximum, effective, and instantaneous (last two); s, s_T, s_g, heat assimilations of the medium, the grains, and the gas; f_0, relative period of contact between the surface and the bubbles; φ, frequency of packet replacement; g, gravitational acceleration; H, height of the fluidized bed; b, δ, thickness of the bed or of the gas layer or heater; d, grain diameter; ν, kinematic viscosity of the gas; τ, τ^*, τ_{min}, τ_0, η, time — current, characteristic, time required to reach the minimum temperature, contact time between packet and heater, and dimensionless time; θ, θ_0, $\hat{\theta}$, temperature of heater, initial temperature, and dimensionless temperature; T, temperature of the medium (packet); q, q_0, thermal flux and heat released per unit surface of the heater; M, parameter representing the relative inertia of the heater.

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